Advanced Econometrics
Homework 4

May 15, 2022

Q1

Consider the following model

$$
\begin{aligned}
y_i &= \beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki} + u_i \\
&= x_i \beta + u_i
\end{aligned}
$$

where

$$
\begin{aligned}
u_i^2 &= \gamma_0 + w_{1i}^2 \gamma_1 + w_{2i}^2 \gamma_2 + \epsilon_i \\
&= w_i \gamma + \epsilon_i
\end{aligned}
$$

$$
\begin{aligned}
E(u_i | w_i) = 0 \\
E(\epsilon_i | w_i) = 0
\end{aligned}
$$

1. Is the OLS estimator of β

   $$\hat{\beta}^{ols} = \left(\sum_{i=1}^{n} x_i' x_i \right)^{-1} \sum_{i=1}^{n} x_i' y_i$$

   efficient in the Gauss Markov sense?

1. How would you go about estimating γ? Describe the procedure and sketch a proof of consistency of the resulting estimator.

Q2

Suppose you have the following model

$$y_i = \beta_0 + \beta_1 x_i + u_i \tag{1}$$
$$x_i = \gamma_0 + \gamma_1 z_i + \eta_i, \tag{2}$$

with $cov(u_i, \eta_i) > 0$ and $E[z_i \eta_i] = 0$.

1. What is the most you can say about $E[x_i u_i]$?
2. Derive the reduced form of y_i.
3. Propose a consistent estimator of ξ, the coefficient of z_i in the reduced form.
4. Let $\hat{\gamma}_1$ be the OLS of a regression of x_i on a constant and z_i, i.e.,
$$\hat{\gamma}_1 = \frac{\sum_{i=1}^n (z_i - \bar{z})(x_i - \bar{x})}{\sum_i^n (z_i - \bar{z})^2}.$$
5. Can you derive the probability limit of $\hat{\xi}/\hat{\gamma}_1$, where $\hat{\xi}$ is the estimator you derived in point 3?

Q3

Consider the following model for a panel dataset

$$y_{it} = \alpha_i + S_t + x'_{it}\beta + \varepsilon_{it}, \quad t = 1, \ldots, T$$

where α_i and S_t are unobserved and

$$E(\varepsilon_{it} | x_{i1}, \ldots, x_{iT}, \alpha_i, S_t) = 0, \quad E(x'_{it} S_t) = 0,$$

Under this setup answer the following questions providing the reasoning behind them.

1. Does the within estimator
$$\hat{\beta}^w = \left(\sum_{i=1}^n \sum_{t=1}^T \ddot{x}'_{it} \ddot{x}_{it} \right)^{-1} \left(\sum_{i=1}^n \sum_{t=1}^T \ddot{x}'_{it} \ddot{y}_{it} \right)$$
where
$$\ddot{y}_{it} = y_{it} - \frac{1}{T} \sum_{i=1}^T y_{it} \quad \ddot{x}_{it} = x_{it} - \frac{1}{T} \sum_{i=1}^T x_{it}$$
is consistent and unbiased for β?

2. Is the first difference estimator

$$\hat{\beta}^w = \left(\sum_{i=1}^{n} \sum_{t=1}^{T} \Delta x'_{it} \Delta x_{it} \right)^{-1} \left(\sum_{i=1}^{n} \sum_{t=1}^{T} \Delta x'_{it} \Delta y_{it} \right)$$

where

$$\Delta y_{it} = y_{it} - y_{i,t-1} \quad \Delta x_{it} = x_{it} - x_{i,t-1}$$

consistent and unbiased for β?

3. If in addition to the conditions above you may assume that $E(\alpha_i x_{it}) = 0$, how will the answer to the previous two points change?