Homework 1

Advanced Econometrics

Due on Wednesday, March 27

Answer the questions below in whatever format you prefer (paper or electronic) as long it contains the code and the answer to all questions in a form that is intelligible to humans. If you choose to answer electronically (which is good for the environment), you should send the document by email to giuseppe.ragusa@gmail.com before the end of Wednesday's lecture. The subject of the email should read "AdvMetrics - Homework 1". Email with different subject lines will not be considered and you will not receive credit for your work.

The asterisk(s) in front of the questions qualify the degree of difficulty of the problems. The higher the number of asterisks the harder the question. If you cannot answer the hardest don't worry too much — it is optional.

1. [*] Use `R` to generate $n = 200$ from the following model

   $$y_i = \beta_0 + \beta_1 x_i + u_i, \quad n = 1, \ldots, 200$$

   where x_i and u_i are independent and (u_i, x_i) is independent of (u_j, x_j) for $i \neq j$;

   $$u_i \sim N(0, 1), \quad x_i \sim N(0, 1)$$

   and $\beta_0 = 0.5$ and $\beta_1 = 0.3$.

2. [*] Using the data generated in the previous model to estimate the β_0 and β_1 using OLS.

3. [**] Calculate the variance of $\hat{\beta}_1$ that is appropriate for the assumption of the model. (*Note: You can use the output from* `lm` *and the* `summary` *method, but you will get extra credit if you try to calculate the variance without relying on it. Of course, you can use R to check whether your calculations are correct.)*

4. Construct a 90% confidence interval for β_1.

5. [**] Since you generated the data yourself, you know which assumptions hold for the model above. Answer the question below providing a brief (1 line max) justification:

   (a) Is the OLS estimator unbiased for β_1? (Brief justification)

(b) Is the model conditionally homoschedastic?

(c) Is $\hat{\beta}_1$ estimating the casual effect or is it estimating the linear projection coefficient?

6. [*] Suppose that instead of running a regression of y_i on x_i, you run the regression of x_i and y_i, that is you switch the dependent and independent variables

$$x_i = \gamma_0 + \gamma_1 y_i + \eta_i.$$

Report your estimates of γ_1.

7. [****] What is γ_1 estimating, i.e. can you give the probability limit of $\hat{\gamma}_1$?

8. [For the brave] Find the asymptotic distribution of the estimator defined as $\hat{\xi} = 1/\hat{\beta}_1$ where $\hat{\beta}_1$ is the OLS estimator of point [2]. *[Hint: use the Delta Method.]*